solve $\int \sin^n x\,dx = \int \sin^{(n-1)} x \sin x\,dx$

Let $u = \sin^{n-1} x \Rightarrow du = (n-1)\sin^{n-2} x \cos x\,dx$

$dv = \sin x\,dx \Rightarrow v = -\cos x$

The rule is

$$\int u\,dv = u.v - \int v\,du$$

$$\Rightarrow \int \sin^n x\,dx = -\cos x \sin^{n-1} x - \int -\cos x\,(n-1)\sin^{n-2} x \cos x\,dx$$

$$\Rightarrow \int \sin^n x\,dx = -\cos x \sin^{n-1} x + (n-1)\int \sin^{n-2} x \cos^2 x\,dx$$

$$\int \sin^n x\,dx = -\cos x \sin^{n-1} x + (n-1)\int \sin^{n-2} x\,(1 - \sin^2 x)\,dx$$

$$\int \sin^n x\,dx = -\cos x \sin^{n-1} x + n-1\int (\sin^{n-2} x - \sin^n x)\,dx$$

$$\int \sin^n x\,dx = -\cos x \sin^{n-1} x + n-1\int \sin^{n-2} x\,dx - (n-1)\int \sin^n x\,dx$$

$$(n-1)\int \sin^n x\,dx + \int \sin^n x\,dx = -\cos x \sin^{n-1} x + (n-1)\int \sin^{n-2} x\,dx$$

$$n\int \sin^n x\,dx \cancel{- \int \sin^n x\,dx} \cancel{+ \int \sin^n x\,dx} = -\cos x \sin^{n-1} x + (n-1)\int \sin^{n-2} x\,dx$$

$$n\int \sin^n x\,dx = -\cos x \sin^{n-1} x + (n-1)\int \sin^{n-2} x\,dx \quad \boxed{\div n}$$

$$\int \sin^n x\,dx = \frac{-1}{n}\cos x \sin^{n-1} x + \frac{(n-1)}{n}\int \sin^{n-2} x\,dx \;.\;\square$$