The Fundamental Theorem of Calculus

The fundamental theorem of calculus has two parts:

Theorem (Part I). Let f be a continuous function on $[a, b]$ and define a function $g\colon [a, b] \to \mathbf{R}$ by

$$g(x) := \int_a^x f.$$

Then g is differentiable on (a, b), and for every $x \in (a, b)$,

$$g'(x) = f(x).$$

Theorem (Part II). Let f be a continuous function on $[a, b]$. Suppose that F is continuous on $[a, b]$ and that $F' = f$ on (a, b). Then

$$\int_a^b f = F(b) - F(a).$$