Some various examples of Integrals-

* $\int \frac{\cot x}{\ln(\sin x)}\,dx$, let $u = \ln(\sin x)$

$du = \frac{1}{\sin x}\cdot \cos x\,dx$

$du = \cot x\,dx$.

$= \int \frac{du}{u} = \ln u + C$

$= \ln(\ln(\sin x)) + C$.

* $\int \frac{e^x - e^{-x}}{e^x + e^{-x}}\,dx$, let $u = e^x + e^{-x}$

$du = e^x - e^{-x}\,dx$

$= \int \frac{1}{u}\,du = \ln u + C$

$= \ln(e^x + e^{-x}) + C$.

$= \int \tanh x\,dx = \ln(\cosh x) + C$.

$\tanh x = \frac{\sinh x}{\cosh x}$

* $\int \sqrt{\frac{a+x}{a-x}}\,dx = \int \sqrt{\frac{a+x}{a-x} * \frac{a+x}{a+x}}\,dx = \int \frac{a+x}{\sqrt{a^2-x^2}}\,dx$

$= \int \frac{a}{\sqrt{a^2-x^2}}\,dx + \int \frac{x}{\sqrt{a^2-x^2}}\,dx = a\sin^{-1}\frac{x}{a} - \frac{1}{2}\int (a^2-x^2)^{-\frac{1}{2}}(-2x)\,dx$

$= a\sin^{-1}\frac{x}{a} - \sqrt{a^2-x^2} + C$

* $\int \frac{1}{e^x - 1}\,dx \times \frac{e^x}{e^x}$

$= \int \frac{e^x}{e^x(e^x-1)}\,dx$, let $z = e^x \rightarrow dz = e^x dx$

$= \int \frac{1}{z(z-1)}\,dz$.

* $\int \frac{1}{\text{quadratic}}\,dx$

① Make the coefficient of x^2 "one" by taking it outside.
② Complete the square in terms containing x by adding & subtracting the square of half the coefficient of x.
③ use the standard form to calculate it.

e.g. $\int \frac{dx}{2x^2-2x+1} = \frac{1}{2}\int \frac{1}{x^2-x+\frac{1}{2}}\,dx = \frac{1}{2}\int \frac{dx}{x^2-x+\frac{1}{4}-\frac{1}{4}+\frac{1}{2}}$

$= \frac{1}{2}\int \frac{1}{(x-\frac{1}{2})^2+\frac{1}{4}}\,dx = \frac{1}{2}\int \frac{1}{(x-\frac{1}{2})^2+(\frac{1}{2})^2}\,dx$

$= \frac{1}{2}\cdot\frac{1}{\frac{1}{2}}\tan^{-1}\left(\frac{x-\frac{1}{2}}{\frac{1}{2}}\right)+C = \tan^{-1}(2x-1)+C$

$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\tan^{-1}\frac{x}{a}+C$

* $\int \frac{\sin^{-1}x}{x^2}\,dx$, let $x=\sin\theta$

$dx=\cos\theta\,d\theta$

$= \int \frac{\theta}{\sin^2\theta}\cdot\cos\theta\,d\theta = \int \theta(\csc\theta\cot\theta)\,d\theta$

let $u=\theta$, $du=d\theta$

$dv=\csc\theta\cot\theta\,d\theta$, $v=-\csc\theta$

$= -\theta\csc\theta + \int \csc\theta\,d\theta = -\theta\csc\theta + \ln|\csc\theta-\cot\theta)+C$

$= -\frac{\theta}{\sin\theta} + \ln\left(\frac{1-\cos\theta}{\sin\theta}\right)+C = -\frac{\sin^{-1}x}{x} + \ln\left|\frac{1-\sqrt{1-x^2}}{x}\right|+C$